International Journal of Theoretical Physics, Vol. 26, No. 3, 1987

Physical Experiments and Computation

Andrzej Posiewnik[1]

Received November 13, 1986

A description is given of general (preparation–measurement) physical experiments in the language of Scott's mathematical theory of computation.

> And all I was doing was hoping that the computer-type of thinking would give us some new ideas, if any are really needed.
> Richard P. Feynman

Following Ludwig (1983), we shall use the expression *fundamental domain of a physical theory* to denote those facts that can be described without the considered theory itself and are mapped into the mathematical formalism dealing with the logicomathematical structure of the theory.

We shall seek to describe the fundamental domain for the case of experiments with a single microsystem. Our purpose is to look very carefully at the tacit connotations and implications contained in the customary use of the term "physical experiment," and to find out exactly how to give expressions to those connotations and implications by means of Scott's theory of computation (Scott, 1981). This paper is a continuation of our previous papers (Posiewnik, 1985, 1986).

Without restricting ourselves to any particular example of a physical experiment, we shall proceed to give as general a description of a physical experiment as possible. Usually we perform the following types of experiments: by certain well-defined macroscopic procedures we prepare a state of affairs for study, and at some later time, according to other well-defined procedures, we interact with the state of affairs so obtained to obtain information about it. We will assume that each experiment that one is

[1]Institute of Theoretical Physics and Astrophysics, University of Gdańsk, 80-952 Gdańsk, Poland.

0020-7748/87/0300-0239$05.00/0

interested in performing can be thought of as being on a fixed individual microsystem. According to Piron (1976), every microsystem is in a definite (pure) state.

Let S denote the abstract set of all (pure) microstates of our system.

A state of the microsystem depends upon how we prepared the system.

Each experiment has a finite formal description (p, O) consisting of an initial "preparation procedure" p [precontrolling in Finkelstein's (1979) sense] followed by an "observation" (postcontrolling) O. We define a *preparation procedure* as an ordered, physically realizable list of instructions.

A single measurement on a quantum microsystem hardly gives us any information at all. Therefore we have to create a large number of identical microstates of a given system and perform the same measurement on each of them.

But what does it mean to say that the microstates are indeed "identical"? We have no way of controlling the large number of microscopic constituents of the preparing apparatus. Therefore, we can never arrange the apparatus in such a way that we can confidently assert (the word "assert" is used here not in the sense of "predict," but in the sense of making an *a priori* true statement) that our system emerging from the setup is in a *given* (pure) microstate. The only thing one can say in this situation is that a family of microstates was prepared according to the same macroscopic preparation procedure p. Therefore, one must always state that a preparation process p maps a definite set of precontrolling parameters into a set $P \subseteq S$ of microstates. The set P depends on certain technical facts about the macroscopic preparation apparatus in which microsystems can be produced in large numbers.

If $s \in S$ is a microstate and p is a preparation procedure, then $s \in P$ means that the microstate s has been prepared according to the preparation procedure p. It is clear from this discussion that the most fundamental entities that are useful for our formalism are the subsets of the set S of all microstates of our system. It is true that one can always assume the existence of points in S, but they must be considered to exist only in the sense of some limiting process.

In the mathematical framework (syntax) of our theory we shall express the notion of *the prepared state* in the following way:

Definition 1. Let $\mathscr{P}$ be a nonempty system of subsets of the set S of microstates, where:

(i) $S \in \mathscr{P}$

(ii) If $P_1, P_2, P_3 \in \mathscr{P}$ and $P_3 \subseteq P_1 \cap P_2$, then $P_1 \cap P_2 \in \mathscr{P}$

The elements of the system $\mathscr{P}$ will be called *the prepared states.*

The above conditions describe a well-defined set-theoretic "formal structure" and, as such, they require no further justifications. However, the heuristics are as follows. The elements of $\mathscr{P}$ represent definite preparation procedures. Condition (i) means that there is a common preparation procedure for each state of our system. The procedure may be described in the following way: Chose as you want, at random or not, one of the preparators for the system and execute a preparation procedure at your own choice.

Condition (ii) is weaker (and it seems to us more reasonable as well) than the Ludwig (1983) axiom AS 1.2 for the preparation procedures. Our condition states that if the set of states prepared according to both preparation procedures P_1 and P_2 (that is, the set $P_1 \cap P_2$) contains a set of states prepared according to a preparation procedure P_3, then $P_1 \cap P_2$ is a possible preparation procedure.

Now let us define a notion of *the experimental state.* The idea behind the concept is the following: if we want to explain the experimental meaning of a particular (pure) microstate to an experimenter in practice we can give a sequence $P_0 \supseteq P_1 \supseteq P_2 \supset \cdots \supseteq P_n \supseteq \cdots$ of prepared states of increasing refinement. The "better and better' prepared states P_n are partial approximations to an "ideal limit."

The ideal limit, which in some situations is exactly the sought for (pure) microstate, is an abstraction which must be regarded purely as a device to make our theories easier to handle. If we have two "convergent" sequences of prepared states

$$P_0 \supseteq P_1 \supseteq \cdots \supseteq P_n \supseteq \cdots$$

$$Q_0 \supseteq Q_1 \supseteq \cdots \supseteq Q_n \supseteq \cdots$$

we say that the two sequences are equivalent, i.e., determine the same "*limit,*" iff

$$\bigwedge_i \bigvee_k \text{ such that } P_k \subseteq Q_i$$

$$\bigwedge_k \bigvee_i \text{ such that } Q_i \subseteq P_k$$

This is an equivalence relation and the "ideal limits" can be identified with the equivalence classes.

From the above construction we can abstract a bit more general a definition of *the experimental state.*

Definition 2. The experimental state (the "ideal limit" of "better and better" prepared states) over a system $\mathscr{P}$ of prepared states is a subfamily $x \subseteq \mathscr{P}$ where

(i) $S \in x$

(ii) $(P_1, P_2 \in x) \Rightarrow (P_1 \cap P_2 \in x)$

(iii) $(P_1 \in x \text{ and } P_1 \subseteq P_2 \in \mathcal{P}) \Rightarrow P_2 \in x$

The family of all experimental states over the system $\mathcal{P}$ is written as $\bar{\mathcal{P}}$.

The careful reader acquainted with Scott's (1981) theory of neighborhood systems may notice that our definitions of the prepared and the experimental states are notional counterparts of Scott's neighborhood systems and of (ideal) elements of the systems. This does not surprise us, because if we take seriously Fredkin's idea of "physics as information" (see Wheeler, 1982) we have to agree that the same structures should arise in the foundations of physics as in the foundations of the mathematical theory of computation.

Now we will give a mathematical description of the second part of a physical experiment—i.e., observation (measurement, postcontrolling). It will not be a precise mathematical description of a macroscopic measurement apparatus. Instead, we shall only assume that there exists an objective characterization of the mode of operation of the apparatus. In the precontrolling phase of a physical experiment we prepare a microsystem in some (pure) microstate s. The only true thing we can say about the input state s, is that it was prepared according to a preparation procedure p, so it belongs to a prepared state P; $s \in P$. After performing the measurement we obtain a number on the numerical scale of the measuring apparatus. Because the input information is in some sense "fuzzy," the postcontrolling number is "fuzzy," too.

Definition 3. Let $\mathbb{R}$ be the real line. Let $\mathcal{N}$ be the set of nonempty open intervals with rational end points plus the whole $\mathbb{R}$. It is easy to show that the system satisfies the conditions of Definition 1.

The elements of the above system will be called *the numerical states.*

Taking the above into account, we can state that on the most primitive level one can represent a (numerical) measurement procedure as a relation from a system $\mathcal{P}$ of the prepared states to the system $\mathcal{N}$ of the numerical states. If f denotes the relation, then PfN $(P \in \mathcal{P}, N \in \mathcal{N})$ means that for each input (micro) state from the prepared state P, after the measurement we obtain as an output a number from the interval N. (The relation f is *fixed* for a definite measuring apparatus.)

It is evident that if one wishes to construct a mathematical formalism for a description of a fundamental domain of a physical theory—a formalism which can always be enriched in structure without changing its basic form—then one must start from the weakest assumptions. The minimal conditions on the *measurement relation* are as follows:

(i) If the only fact we know is that the input microstate belongs to the set S of all microstates of our system, then we can only assert that the result of the measurement operation f lies somewhere in R.

So we can write

$$Sf\mathbb{R}$$

(ii) Suppose that for each microstate from a prepared state P the measured values belong to a numerical state N_1 and in another series of postcontrolling we conclude that for the same input prepared state P the measured values belong to a numerical state N_2 as well.

Then the result of measurement for prepared state P should belong to the intersection of the sets N_1 and N_2, i.e., $N_1 \cap N_2$:

$$(PfN_1 \text{ and } PfN_2) \Rightarrow (Pf(N_1 \cap N_2))$$

for each $P \in \mathcal{P}$; $N_1, N_2 \in \mathcal{N}$.

(iii) Suppose we know that $P_1 f N_1$ for some $P_1 \in \mathcal{P}$ and $N_1 \in \mathcal{N}$. If we improve the accuracy of the preparation process for the measurement f, i.e., instead of the prepared state P_1 we input a prepared state P_2, $P_2 \subseteq P_1$, and if we degrade the accuracy of N_1 to N_2, $N_2 \supseteq N_1$, then we can still assert $P_2 f N_2$.

For each $P_1, P_2 \in \mathcal{P}$ and $N_1, N_2 \in \mathcal{N}$ such that $P_1 \supseteq P_2$ and $N_2 \supseteq N_1$, if $P_1 f N_1$, then $P_2 f N_2$.

The above minimal conditions on a measurement relation correspond exactly to the Scott's (1981) notion of an approximable mapping.

Now we will prescribe in the language of our formalism some other suitable notions and theorems of Scott's theory.

Definition 4. Let P be a prepared state, $P \in \mathcal{P}$. The *finite experimental state* determined by P is defined by

$$\uparrow P = \{Q \in \mathcal{P} \colon P \subseteq Q\}$$

Of course a finite experimental state is at the same time an experimental state. The finite experimental states are "dense" in the set $\bar{\mathcal{P}}$ of all experimental states.

For each experimental state x

$$x = \bigcup \{\uparrow P \colon P \in x\}$$

Any experimental state is uniquely determined by its finite approximations, i.e., is a certain type of "limit" of finite experimental states.

Definition 5. Let x and y be experimental states. We say that x approximates y iff $x \subseteq y$.

Proposition. Let $\mathcal{P}$ be a system of the prepared states and $\mathcal{N}$ a system of the numerical states.

A measurement relation $f: \mathscr{P} \to \mathscr{N}$ always determines a function $\bar{f}: \bar{\mathscr{P}} \to \bar{\mathscr{N}}$ between the experimental states by virtue of the formula

$$\bar{f}(x) = \left\{ N \in \mathscr{N}: \bigvee_{P \in x}, PfN \right\}$$

for all $x \in \bar{\mathscr{P}}$.

Conversely, this function uniquely determines the original relation by the equivalence

$$PfN \quad \text{iff} \quad N \in \bar{f}(\uparrow P) \qquad \text{for all} \quad P \in \mathscr{P}, \quad N \in \mathscr{N}$$

The measurement functions are always monotone, i.e.,

$$(x \subseteq y) \Rightarrow (\bar{f}(x) \subseteq \bar{f}(y))$$

for $x, y \in \bar{\mathscr{P}}$.

Two measurement functions $\bar{f}_1: \bar{\mathscr{P}} \to \bar{\mathscr{N}}$ and $\bar{f}_2: \bar{\mathscr{P}} \to \bar{\mathscr{N}}$ are identical as relations iff

$$\bar{f}_1(x) = \bar{f}_2(x) \qquad \text{for all} \quad x \in \bar{\mathscr{P}}$$

Topology. Given a system $\mathscr{P}$ of prepared states, define for $P \in \mathscr{P}$

$$[P] = \{x \in \bar{\mathscr{P}}: P \in x\}$$

Then the sets $[P]$ for $P \in \mathscr{P}$ make $\bar{\mathscr{P}}$ into a topological space.

The measurement functions $\bar{f}: \bar{\mathscr{P}} \to \bar{\mathscr{N}}$ are exactly *the continuous functions between these spaces.*

CONCLUSION

The mathematical theory of computation in Scott's formulation forms a suitable mathematical framework (syntax) of the fundamental domain of preparation–observation experiments with single microsystems.

Other consequences of the above representation will be studied in subsequent papers.

REFERENCES

Finkelstein, D. (1979). The physics of logic, in *The Logico-Algebraic Approach to Quantum Mechanics*, vol. II, C. A. Hooker ed., D. Reidel, Dordrecht.

Ludwig, G. (1983). *Foundations of Quantum Mechanics*, Springer, Berlin.

Piron, C. (1976). *Foundations of Quantum Physics*, Benjamin, Reading, Massachusetts.

Posiewnik, A. (1985). On some definition of physical state, *International Journal of Theoretical Physics*, **24**, 135.

Posiewnik, A. (1986). Dynamical transformations and information systems, *International Journal of Theoretical Physics*, **25**, 891.

Scott, D. (1981). Lectures on a Mathematical Theory of Computation, Technical Monograph PRG-19, Oxford.

Wheeler, J. A. (1982). The computer and the universe, *International Journal of Theoretical Physics*, **21** (6/7).